**11.1 Permutations and Combinations**.  
Objective: To count permutations and combinations

*Scenario*: *You go to a deli. There are 4 types of meat (Turkey, Ham, Roast Beef, and Pastrami) and 3 types of bread (White, Wheat, and Rye). How many different sandwich combinations do you have to choose from?*

Method 1, **Tree Diagram**: charting every possible combination of multiple sets of data as a web



* Count the number of options on the final column (12 combination)

*Method 2***, Fundamental Counting Principle**;

* **Two Events**: If one event can occur in m ways and another event can occur in n ways, then the number of ways that both events can occur is **m\*n**

*Example*: 4 meat choices, 3 bread choices; (4)(3)=12 combinations

* **Three or More Events**: If three events can occur in m, n, and p ways, then the number of ways that all three events can occur is **m\*n\*p**. And so on for > 3 events

*Example*: 4 meat choices, 3 bread choices, 2 cheese choices

(4)(3)(2) = 24 combinations

*Examples:*

*Each event can occur in the given number of ways. Find the number of ways all the events can occur.*

*a) Event 1: 3 ways b) Event 1: 2 ways*

*Event 2: 7 ways Event 2: 4 ways*

*Event 3: 5 ways*

*Police use photographs of various facial features to help witnesses identify suspects. One basic identification kit contains 195 hairlines, 99 eyes and eyebrows, 89 noses, 105 mouths, and 74 chins and cheeks.*

a) The developer of the identification kit claims that it can produce billions of different faces. Is this correct?

b) A witness can clearly remember the hairline and eyes and eyebrows of a suspect. How many different faces can be produced with this information?

*The standard configuration for a New York license plate is 3 digits followed by 3 letters.*

a) How many different license plates are possible if digits and letters can be repeated?

b) How many different license plates are possible if digits and letters cannot be repeated?

**Factorial:** n!; the product of all integers from 1 to n

denoted by ! (ex: 3! = 3\*2\*1 = 6)

Evaluate the factorial:

a) 4! b) 6!

c) 13! d) 1!

**Permutation**: an ordering of n objects

(Each different order/combination is a new permutation)

* Fundamental counting principle can be used to determine the number of permutations of n objects

*Example*: *How many permutations are there for the letters A, B, and C?*

**Number of Permutations of n Distinct Objects**;

**n! = n\*(n – 1)\*(n – 2)\*…\*3\*2\*1**

*Example*: *Twelve skiers are competing in the final round of the Olympic freestyle skiing aerial competition*

a) In how many different ways can the skiers finish the competition?

b) In how many different ways can 3 of the skiers finish first, second, and third to win gold, silver, and bronze?

**Permutations of n Objects Taken r at a Time**;

The number of permutations of r objects taken from a group of n distinct objects is denoted by nPr and is given by*:*

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*Find the number of permutations:*

10P6 5P1

*Example: You are considering 10 different colleges. Before you decide to apply to the colleges, you want to visit some of them. In how many ways can you visit,*

*a) 6 of the colleges? b) 4 of the colleges?*

**Permutations with Repetition**;

The number of distinguishable permutations of n objects where one object is repeated q1 times, another is repeated q2 times, and so on is:

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*Example:*

*Find the number of distinguishable permutations of the letters in the word.*

*a) PENCIL b) LETTER*

**Combination**: a selection of r objects from a group of n objects where the order is not important (ex: hand of cards)

**Combinations of n Objects Taken r at a Time**;

The number of combinations of r objects taken from a group of n distinct objects is denoted by nCr and is given by:

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n = number to total objects

r = number of objects taken from the group

*Examples:*

*Find the number of combinations of n objects taken r at a time*

*a) n = 8, r = 2 b) n = 6, r = 5*

**Multiple Events:**

Event A and Event B → Multiply

Event A or Event B → Add

*Example:*

*A restaurant serves omelets that can be ordered with any of the ingredients shown.*

*Vegetarian Meat*

*Green Pepper Ham*

*Red Pepper Bacon*

*Onion Sausage*

*Mushroom Steak*

*Tomato*

*Cheese*

a) Suppose you want exactly 2 vegetarian ingredients and 1 meat ingredient in your omelet. How many different types of omelets can you order?

b) Suppose you can afford at most 3 ingredients in your omelet. How many different omelets can you order?

An amusement park has 20 different rides. You want to ride at least 16 of them. How many different combinations of rides can you go on?

The movie theater has 12 movies showing. You want to go to at least 3 of the movies. How many different combinations of movies can you attend?

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